MATHEMATICS ENRICHMENT CLUB.
Problem Sheet 9, June 27, 2016

1. There is a positive integer A. Two operators are allowed: increasing this number by 9 and deleting a digit equal to 1 from any position. Is it always possible to obtain $A+1$ by applying these operations several times?

2. A triangle was drawn on a plane such that the sum of the x-coordinates and the sum of the y-coordinates of this triangle are 12 and 96 respectively. Find the coordinate of the point of interception of all three medians of this triangle (The median of a triangle is a line segment joining a vertex to the midpoint of the opposing side).

3. If a, b, c are the roots of the equation $x^3 + 2x = 1$. Then find the value of

$$(a^3 + a^2 + a) + (b^3 + b^2 + b) + (c^3 + c^2 + c).$$

4. Four digits are selected from the set $\{1, 2, 3, 4, 5\}$ to form a 4-digit number. Find the sum of all possible permutations.

5. Use each of the nine digits $1, 2, 3, \ldots, 9$ exactly once to form prime numbers whose sum is as small as possible.

6. $77!$ is perfectly divisible by $(6!)^n$. Find the maximum value of n.

Senior Questions

1. Given that $n > 1$ is prime if and only if the remainder of $(n-1)!$ is $n-1$ divided by n. Solve the system of simultaneous equations:

$$\begin{cases} p! + 1 = (2p+1)^2 \\ q! + 1 = (10q + p - 4)^2 \end{cases}.$$

2. Solve

$$\int_{\frac{\sqrt{2}}{2}}^{1} \frac{1}{x^5\sqrt{4x^2-1}}\,dx.$$

3. Let C be a right angle in triangle ABC. On legs AC and BC the square $ACKL$, $BCMN$ are constructed outside of the triangle. If CE is an altitude of the triangle; see diagram below. Prove that LEM is right angle.