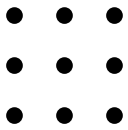
MATHEMATICS ENRICHMENT CLUB.
Problem Sheet 11, August 1, 2016

1. Let $29x + 30y + 31z = 366$, where x, y and z are non-negative integers. Find the least value of $x + y + z$.

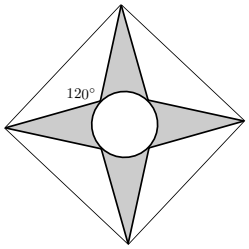
2. A 3×3 grid of dots is shown below. What is the minimum number of squares required to separate each dot from the rest?



3. Find the largest integer n, such that $n^3 + 100$ is divisible by $n + 10$.

4. How many five-digit natural numbers are divisible by 5 and the sum of its digits is also divisible by 5?

5. A circle is inscribed in a diamond of side length 1, such that the neighboring corners of the diamond intersect the circle at 120°; as shown below. Find the area of the shaded region.



6. Consider the set of 2016 integers of the form $1!, 2!, 3!, \ldots, 2015!, 2016!$. Find the maximum number of integers from this set, such that their product forms a perfect square; For example, $1! \times 3! \times 4! = 144 = 12^2$ is a perfect square.

Senior Questions

1. Positive integers $a < b < c$ are such that $b + a$ is a multiple of $b - a$ and $c + b$ is a multiple of $c - b$. If a is a 2011-digit number and b is a 2012-digit number, exactly how many digits does c have?

2. $$\sum_{k=1}^{\infty} \frac{k}{5^k} =$$

3. In a convex quadrilateral $ABCD$, $AB = 10$, $BC = 14$, $CD = 11$ and $DA = 5$. Determine the angle between its diagonals.