# MATHEMATICS ENRICHMENT CLUB.
## Problem Sheet 3, May 20, 2014[1]

1. When multiplying two whole numbers a student by mistake reduced the tens digit in the answer by 7. She checked her answer by dividing it by the smaller factor, obtaining the quotient 48 and the remainder 17. Find the two factors.

2. Find all integers $x, y, z$ such that
$$\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=\frac{1}{x+y+z}.$$

3. $N$ is a number less than 500 with three distinct digits, none of them 0. Five different numbers can be obtained by changing the order of the digits of $N$. The arithmetic mean of these five numbers is equal to $N$. Find $N$.

4. We only know that the password of a safe consists of 7 different digits. The safe will open if we enter 7 different digits, and one of them matches the corresponding digit of the password. Can we open this safe in less than 7 attempts?

5. A regular polygon with $n$ sides is inscribed in a circle. If $A$, $B$, $C$ and $D$ are four successive vertices of the polygon then the length of $AD$ equals the side of the polygon plus the radius of the circle. Find all possible values of $n$.

6. A Rubik's cube is a puzzle of a $3\times 3\times 3$ cube divided into unit sized cubes centred on a pivot which allows each face of 9 smaller cubes to be rotated. Each exposed face is coloured so that the faces of a solved Rubik's cube consist of one colour. How many possible permutations of a Rubik's cube are there?

### Senior Questions

1. Let $f$ and $g$ be functions differentiable at $x_0$. Show that $fg$ and $f/g$ (if additionally $g(x_0)\neq 0$) are also differentiable at $x_0$.

2. Let $g$ be differentiable at $x_0$ and $f$ differentiable at $g(x_0)$. Show that $f\circ g$ is differentiable at $x_0$.

3. Suppose $f$ and $g$ are functions and that $g$ is non-zero and continuous at $x_0$. Given that $fg$ and $f/g$ are differentiable at $x_0$, show that $f$ is also.

[1]Some problems from UNSW's publication *Parabola*, and the *Tournament of Towns in Toronto*