MATHEMATICS ENRICHMENT CLUB.[1]
Problem Sheet 9, July 23, 2013

1. The sequence $a_1, a_2, a_3, \ldots$ is arithmetic. If $a_1 = 10$ and $a_{a_2} = 100$ what is $a_{a_{a_3}}$?

2. We play a game in which we try to get from one number to another. Each move we can replace the natural number n with ab if $a + b = n$ and a and b are both natural numbers. Can we get to 2001 from 22 in any number of moves?

3. How many digits does the number 125^{100} have?

4. Commander Keen is standing at the top left corner of an $n \times n$ grid, but wants to get to the bottom right corner. He's only allowed to move to the right, or downwards.
   (a) Draw all the possible paths from the top left to the bottom right if the grid is 2×2.
   (b) How many possible paths are there if the grid is 20×20?
   (c) What about $n \times n$?

5. Each of the six vertices of a regular hexagon are connected to every other vertex using either a red or a blue line. Show that, however this is done, the resulting diagram will always contain either a red or a blue triangle. Show that this is not always the case if we use the vertices of a pentagon.

6. Consider the two sequences $x_0 = 1, x_1 = 1, x_{n+1} = x_n + 2x_{n-1}$ and $y_n = 8n + 1$. Prove that for $n > 1$ these two sequences never have a common term.

Senior Questions

1. Prove that
$$1 \times 3 \times 5 \times \ldots \times (2n-1) = \frac{(2n)!}{2^n n!}.$$

2. By considering $\cos(A + B) + \sin(A - B) = 0$ find the general solution (for θ) of $\cos n\theta + \sin m\theta = 0$.

[1]Some of the problems here come from T. Gagen, Uni. of Syd. and from E. Szekeres , Macquarie Uni.