MATHEMATICS ENRICHMENT CLUB.[1]
Problem Sheet 8, June 20, 2012

1. The last digit of 1997^{1997} is

   (a) 1 (b) 3 (c) 5 (d) 7 (e) 9.

2. The number of positive integers less than 1000 which are divisible neither by 3 nor 4 is

   (a) 582 (b) 499 (c) 500 (d) 7501 (e) None of these.

3. Draw a right triangle ABC with right-angle at C and the sides marked a, b, c.[2]

   (a) Draw the enlargement $A'B'C'$ of ABC by a factor of a.

   (b) On the same diagram draw the enlargement $A''B''C''$ of ABC by a factor of b, lining up $B'C'$ with $A''C''$, so that $A'' = B'$, $C'' = C'$, and A', C' and B'' are collinear, and thus form a new triangle $A'A''B''$.

   (c) Explain why the angle $A'A''B''$ is a right angle.

   (d) What theorem have you just proven and why?

4. Find all positive integer solutions to

$$\frac{1}{x} + \frac{1}{y} = \frac{1}{6}.$$

5. (a) Find the greatest common divisor of $2^{50} + 1$ and $2^{20} + 1$.

   (b) Explain why the greatest common divisor of $2^m + 1$ and $2^n + 1$ is at least 3 if m and n are both odd.

6. (a) Prove that the angle in a semicircle is right-angle.

   (b) Show that if two chords of a circle mutually bisect each other, then they are both diameters.

   (c) Complete the following statement: If a parallelogram is inscribed in a circle then .....

7. (a) Given an equilateral triangle, what is the smallest number of non-overlapping equilateral triangles into which it can be dissected?

   (b) Describe how to dissect it into seven equilateral triangles.

[1] Some of the problems here come from T. Gagen, Uni. of Syd. and from E. Szekeres , Macquarie Uni.
[2] Thanks to Mike Hirschhorn for this problem

Senior Questions

1. Evaluate
$$\lim_{x \mapsto \infty} \frac{3x^2 + \sin(2x^2)}{x^2}$$

2. Evaluate
$$\lim_{x \mapsto 0} \frac{3x^2 + \sin(2x^2)}{x^2}$$