UNSW
THE UNIVERSITY OF NEW SOUTH WALES

Never Stand Still | Faculty of Science | School of Mathematics and Statistics

**MATHEMATICS ENRICHMENT CLUB.**[1]
**Problem Sheet 7, June 9, 2012**

1. In how many ways can 6 boys and 6 girls stand next to each other in a row such that no two boys stand next to each other and no two girls stand next to each other?

2. The notation 5! means $5 \times 4 \times 3 \times 2 \times 1 (= 120)$. How many zeros are there at the end of 1000!.

3. (a) $a, b$ are positive numbers with $a + b = k$. Explain why $ab$ is greatest when $a = b = \frac{k}{2}$.
   (b) Suppose that $x^2 + y^2 = c^2$, find the minimum value of $x^4 + y^4$.

4. (a) Show that there are infinitely many non-zero integers $x, y, z$ such that $2^x + 2^y = 2^z$.
   (b) Show that if $n > 2$ then there are no nonzero integers $x, y, z$ such that $n^x + n^y = n^z$.

5. (Parts b and c require Year 9 and Year 10 Mathematics).
   Let $ABC$ be an isosceles triangle with the base angles $B$ and $C$ being $72°$ and $AB = AC = 4$. The length of the base $BC$, called $x$ is chosen such that a line $CD$ can be drawn, where $D$ lies on $AB$, such that $\angle BDC = 72°$.

   (a) Find a pair of similar triangles and show that $x$ satisfies, $x^2 + 4x - 16 = 0$.
   (b) Use triangle $ABC$ to find $\cos 72°$ in surd form.
   (c) Use triangle $ACD$ to find $\cos 36°$ in surd form.

6. Suppose that two non-parallel straight lines $k$ and $\ell$ meet at a point $P$ which is **not** on the page of my book. Construct a line which would (if $P$ did lie on the page) bisect the angle between the lines and pass through $P$.

7. Let $K, L$ be points on the sides $AB, AD$ respectively of the convex quadrilateral $ABCD$ such that $AK = \frac{1}{3}AB$ and $AL = \frac{1}{3}AD$. Similarly, $M, N$ are points on $CD, CB$ such that $CM = \frac{1}{3}CD$ and $CN = \frac{1}{3}CB$.

   (a) Prove that $KLMN$ is a parallelogram.
   (b) Find the ratio of the area of $KLMN$ to the area of $ABCD$.

[1] Some of the problems here come from T. Gagen, Uni. of Syd. and from E. Szekeres , Macquarie Uni.

## Year 11 Question.

1. Suppose that $m$ and $n$ are positive real numbers. Use trigonometry to find the the maximum value of

$$\frac{m+n}{\sqrt{m^2+n^2}}.$$