MATHEMATICS ENRICHMENT CLUB.[1]
Problem Sheet 13, August 20, 2012

1. Laurie sold two cars for $25 000 each. One he sold at a 20% profit and the other at a 20% loss. How much did he gain or lose ?

2. A pizza has radius z, and height a. What is interesting about it's volume?[2]

3. If a triangle ABC has sides of length a, b, c such that $a^2 + b^2 = c^2$, prove that it must be a right-angled triangle.

4. Without using a calculator, which is larger 31^{24} or 257^{15}.

5. Let $S_n = 2n(2n-1)(2n-2)...(n+1)$. For example, $S_3 = 6 \times 5 \times 4 = 120$.

   (a) What is the power of 2 in the prime factorisatoin of S_n for $n = 2, 3, 4...$?

   (b) Make a conjecture based on(i) and prove it.

6. Without using a calculator, show that

$$\sqrt[3]{5\sqrt{13}+18} - \sqrt[3]{5\sqrt{13}-18} = 3.$$

   (Hint: Let $x = a - b$ and cube.)

7. Let ABC be a triangle and D, E points on AB, BC respectively, and S be the intersection of AE and CD. If $AD = DB$ and $BE : EC = 2 : 1$, find the ratios $CS : SD$ and $AS : SE$.

8. (a) Let P be an interior point in an equilateral triangle ABC. Prove that we can always form a triangle with sides of length AP, BP, CP. (That is, we have to show that the sum of any two of these lengths is larger than the remaining one.)

   (b) Give an example of a triangle and point inside it for which the above result is not true.

[1] Some of the problems here come from T. Gagen, Uni. of Syd. and from E. Szekeres , Macquarie Uni.
[2] This question thanks to Mike Hirschhorn